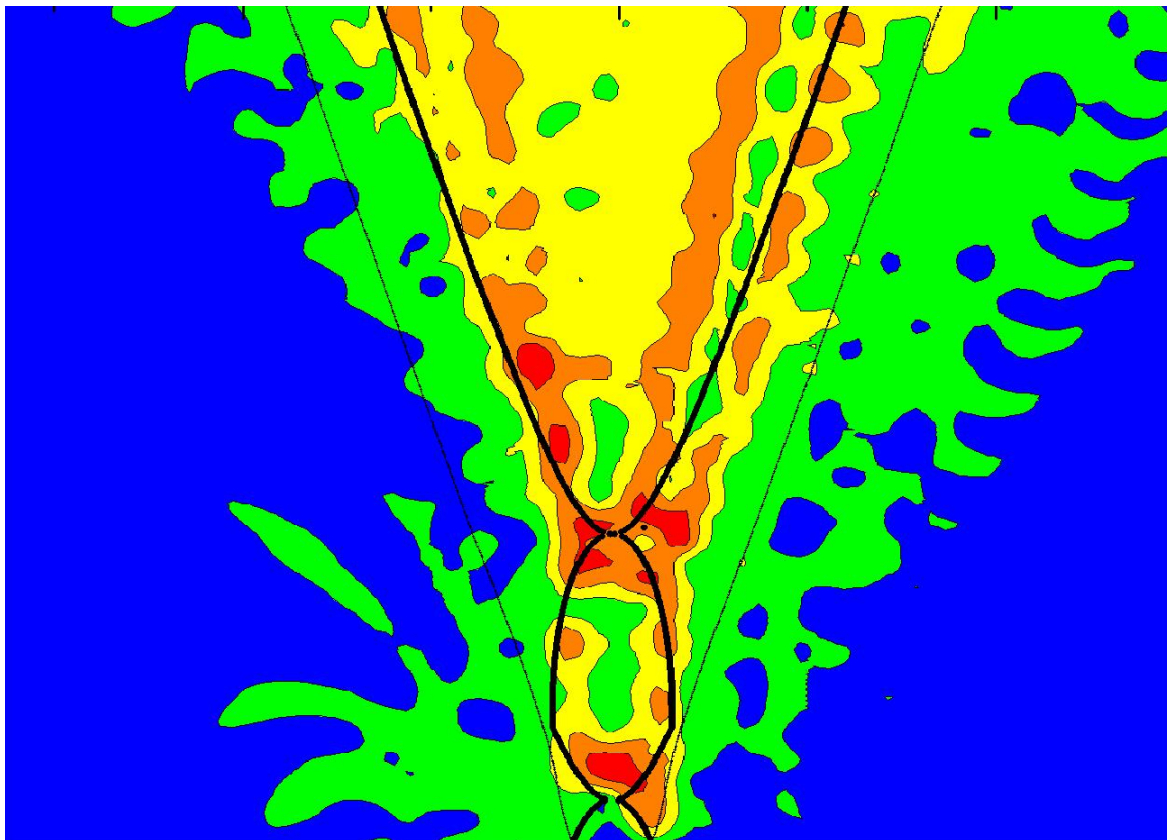


Asymmetry

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Editors

Anastasios Kotronis, Athens, Greece, akotronis@gmail.com, www.akotronismaths.blogspot.com

Kostas Tsouvalas, Lemnos, Greece, kostastsouvalas@windowslive.com

Asymmetry is an electronic mathematical journal which aims at giving the chance to the readers, and the editors themselves, to work on interesting mathematical problems or find information about various mathematical topics. The problems presented may either be original or taken from the existing literature or the web. Attempt will be made towards the precision of the problems' original source. The topics' level is undergraduate and beyond. Readers are encouraged by the editors to submit proposals and/or solutions to proposed problems. Proposals and solutions are preferred to be in L^AT_EX format using what is necessary from the preamble presented in <http://akotronismaths.blogspot.gr/p/asymmetry-electronic-mathematical.html>, must be legible and should appear on separate sheets, each indicating the name of the sender. Drawings must be suitable for reproduction. Proposals should be accompanied by solutions. An asterisk (*) indicates that no solution is available at the time the problem is published. Questions concerning proposals and/or solutions can be sent by e-mail to akotronis@gmail.com.

Problems and Solutions

The source of the problems will appear along with the publication of the solutions

The Problems

V1-1 Evaluate

$$\sum_{n \geq 0} \sum_{k=0}^n \frac{(-4)^k}{(2k+1)} \frac{\binom{n}{k}}{\binom{2k}{k}} x^n$$

for the real values of x that the sum converges.

V1-2 Evaluate

$$\sum_{k \geq 0} \binom{n+k}{2k} \binom{2k}{k} \frac{(-1)^k}{k+1}$$

for $n \in \mathbb{Z}$.

V1-3 Evaluate

$$\sum_{k=0}^n (-1)^k \binom{2n+1}{k} (2n+1-2k)^{2j+1}$$

for $j \in \{0, \dots, n-1\}$.

V1-4 For $\mu > 0$, show that

$$(\ln t)^{1/\mu} \sum_{k=1}^{+\infty} \frac{1}{t^{(2k-1)\mu}} = \frac{\Gamma(1/\mu)}{2\mu} + \mathcal{O}\left((t-1)^{1/\mu}\right) \quad (t \rightarrow 1^+).$$

where Γ denotes the Gamma function.

V1-5 Compute the following limits, if they exist:

1. $\lim_{s \rightarrow +\infty} \frac{1}{\ln s} \int_0^{+\infty} \frac{e^{-\frac{x}{s} - \frac{1}{x}}}{x} dx$ and
2. $\lim_{s \rightarrow +\infty} \left(\int_0^{+\infty} \frac{e^{-\frac{x}{s} - \frac{1}{x}}}{x} dx - \ln s \right).$

V1-6 Show that the equation $ye^y = x$ with $y(0) = 0$ defines a function $y = y(x)$ in $[0, +\infty)$.

For this function, $y(x)$, compute the following limits, if they exist :

1. $\lim_{x \rightarrow +\infty} \frac{y(x)}{\ln x},$
2. $\lim_{x \rightarrow +\infty} \frac{y(x) - \ln x}{\ln(\ln x)},$
3. $\lim_{x \rightarrow +\infty} (y(x) - \ln x + \ln(\ln x)) \frac{\ln x}{\ln(\ln x)}.$

V1-7 Let a_n the sequence defined by $a_n = n(n-1)a_{n-1} + \frac{n(n-1)^2}{2}a_{n-2}$ for $n \geq 3$ and $a_1 = 0, a_2 = 1$.

1. Show that $\lim_{n \rightarrow +\infty} \frac{e^{2n} a_n}{n^{2n+1/2}} = 2\sqrt{\frac{\pi}{e}},$ and
2. compute $\lim_{n \rightarrow +\infty} n \left(\frac{e^{2n} a_n}{n^{2n+1/2}} - 2\sqrt{\frac{\pi}{e}} \right)$ if it exists.

V1-8 If $a_1, \dots, a_n, b_1, \dots, b_m, b \in \mathbb{R}$ with $\sum_{k=1}^n |a_k| + \sum_{k=1}^m |b_k| < b$, evaluate

$$\int_0^{+\infty} \frac{\sin(bx)}{x} \prod_{k=1}^n \frac{\sin(a_k x)}{x} \prod_{k=1}^m \cos(b_k x) dx.$$

V1-9(*) Evaluate $\lim_{n \rightarrow +\infty} \sum_{k=1}^{+\infty} \frac{(-1)^k}{\sqrt[n]{k^k}},$ if it exists.

(if the limit exists and is a real number ℓ , can we make a better estimate than $\sum_{k=1}^{+\infty} \frac{(-1)^k}{\sqrt[n]{k^k}} = \ell + o(1) ?$)